

**FROM THE COLLEGE AND CAREER READY STANDARDS  
TO TEACHING AND LEARNING IN THE CLASSROOM:  
A SERIES OF RESOURCES FOR TEACHERS**

# **BUILDING BLOCKS, LEARNING GOALS, AND SUCCESS CRITERIA: PLANNING INSTRUCTION AND FORMATIVE ASSESSMENT FOR K-8 MATH STANDARDS**

**AUTHORS:**

**GLORY TOBIASON, SANDY CHANG, MARGARET HERITAGE, BARBARA JONES, AND JOAN HERMAN**

National Center for Research on Evaluation, Standards, and Student Testing  
University of California, Los Angeles  
Graduate School of Education & Information Studies



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# INTRODUCTION

This resource is part of a series produced by the Center for Standards and Assessment Implementation (CSAI) to assist teachers and those who support teachers to plan teaching and learning from College and Career Ready Standards (CCRS) for **all** students, including students with disabilities, English learners, academically at-risk students, students living in extreme poverty, and gifted/talented students. The series of resources addresses key shifts in learning and teaching represented in the CCRS. This resource uses the Common Core State Standards (CCSS; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) as an example of CCRS. The processes described in this resource are applicable to all States' CCRS, including the CCSS. The content of this resource is drawn from leading theory and research about learning and formative assessment and from an examination of the CCSS. A section on background reading is included at the end.

This resource is the second in a suite of resources that helps teachers merge content and practice standards of CCRS for Mathematics in daily instruction.<sup>1</sup> The first in this suite, *What's Learned First, What's Learned Together? Developing a Yearlong Plan from the K-8 College and Career Ready Standards for Mathematical Content*, helps teachers arrange and sequence content standards into a Yearlong Plan.<sup>2</sup> This resource assists teachers in moving from a Yearlong Plan to establishing Learning Goals and Success Criteria for daily lesson plans and formative assessment.

This resource is divided into two sections.

- (1) Teachers determine the "Building Blocks" (i.e., series of changes that occur in student thinking or ability) that lead from the previous grade's learning to achievement of the current grade's standards.
- (2) For each Building Block, teachers determine associated Learning Goals and Success Criteria, two key elements of formative assessment.

Teachers who are familiar with the EQuIP rubrics developed by Achieve (2013) will notice several points of connection between this resource and the rubric for evaluating Mathematics Lessons and Units (see "Additional Resources" section for more information). The EQuIP rubrics were designed to help teachers evaluate the quality of instructional materials and may serve as criteria in developing these materials. This series of CSAI resources provides teachers with a process to create instructional materials that address all four dimensions of the EQuIP rubrics (i.e., alignment, key shifts, instructional supports, and assessment). The processes advocated in this specific resource will help teachers meet the following criteria found in the EQuIP rubric:

- Connect the Standards for Mathematical Practice with the CCSS math standards being targeted.
- Develop content through reasoning about the new concepts on the basis of previous understandings.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.

<sup>1</sup> If a State has adopted math CCRS other than the Common Core State Standards in Mathematics (CCSM) and has not specifically identified math practice standards, practice standards have most likely been integrated into the content standards. Teachers in those States should identify and tease out the practice standards from the content standards.

<sup>2</sup> For a copy of the resource, visit: [csai-online.org/resource/448](http://csai-online.org/resource/448).

# BUILDING BLOCKS

The next two sections introduce and describe key concepts in this resource: Building Blocks, Learning Goals, Success Criteria, and Formative Assessment. Tools and processes for teachers to identify and utilize these key concepts are described in later sections of this resource.

## WHAT ARE BUILDING BLOCKS?

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CCRS specify what students should know and be able to do at the end of each grade level. They do not characterize in any detail how student learning progresses from one standard to another. Because the CCRS are substantive and intended to lead to deeper learning, they describe a quantity of learning that is too big for planning daily lessons and too big for formative assessment. For the purpose of planning instruction and formative assessment, teachers need to describe the intermediate steps that lead from achievement of one standard to the next. In this resource, these steps are referred to as Building Blocks, the incremental changes that occur in students' mathematical thinking or ability, as they progress in learning from one standard to the next. These cognitive moves might represent shifts in understanding, increased levels of procedural fluency, or heightened facility in a particular mathematical operation.

Teachers identify Building Blocks by asking, "What are the learnable, lesson-sized 'chunks' of this Standard?" or "What are the incremental learning steps that students need to take on the pathway to achieving this Standard?"

## BUILDING BLOCKS: AN EXAMPLE FROM GRADE 3

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New learning in the Grade 3 CCSS in Mathematics (CCSSM)<sup>3</sup> asks students to "interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each" (3.OA.1). This is the first time students are exposed to the concept of multiplication; the related prior standard from Grade 2 asked them to "use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends" (2.OA.4). To identify the Building Blocks, teachers need to ask: What are key cognitive moves that students need to experience in progressing from the Grade 2 learning ("repeated addition") to the Grade 3 learning ("multiplication")?

Below is a Grade 3 teacher's conceptualization of the Building Blocks that connect the Grade 2 and Grade 3 CCSSM standards. The teacher's thinking that led to her identification of these Building Blocks is made explicit in the annotations.

<sup>3</sup>Henceforth in this document, "Common Core State Standards in Mathematics" is abbreviated as CCSSM.

Building Blocks of a Standard	What the Teacher Was Thinking When Creating the Building Block
<p><b>Block 1</b> Practice repeated addition of objects arranged in rectangular arrays with progressively more rows and columns (beyond 5 rows and 5 columns).</p> <p>EX <math>7+7+7+7</math> and <math>2+2+2+2+2+2+2</math></p>	<p>The idea is that students need to practice this until it's no longer difficult or interesting. This way, they'll be receptive to the idea of multiplication as a "shortcut to repeated addition."</p>
<p><b>Block 2</b> Move between symbolic (<math>2+2+2+2</math>) and concrete (four groups of 2 objects) representations of the same repeated addition number sentence.</p>	<p>Students will need a lot of practice toggling among various representations, and this practice should occur in multiple content areas (science, art, etc.).</p>
<p><b>Block 3</b> Describe repeated addition like <math>2+2+2+2</math> as "the number 2, added four times," and then, "four times 2."</p>	<p>This shift in language is deeply connected to the concurrent, underlying conceptual shift from addition to multiplication.</p>
<p><b>Block 4</b> Extend the learning of Block 3 to include more repeats.</p> <p>EX <math>2+2+2+2+2+2+2</math></p>	<p>Students' practice with different representations should include larger numbers and cross-content connections, as appropriate.</p>
<p><b>Block 5</b> Extend the learning of Block 3 to include more objects in each group.</p> <p>EX <math>7+7+7+7</math></p>	<p>Their practice with different representations should include larger numbers and cross-content connections, as appropriate.</p>
<p><b>Block 6</b> Understand multiplication as a shortcut to repeated addition.</p>	<p>As students work with larger numbers, they can begin to use and notice patterns in the 100's square, addition table, and multiplication table.</p>

A process to create Building Blocks is detailed in the section "From Standards to Building Blocks" on page 8. When teachers clarify the Building Blocks of learning on the pathway from one standard to another, they are better prepared to establish lesson-sized Learning Goals and Success Criteria for instruction and formative assessment; these concepts are described in the next section.

# KEY ELEMENTS OF FORMATIVE ASSESSMENT: LEARNING GOALS AND SUCCESS CRITERIA

## WHAT ARE LEARNING GOALS AND SUCCESS CRITERIA?

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Establishing Learning Goals and Success Criteria are the first steps in planning for formative assessment.

Learning Goals describe what students will learn (not what they will do) during a lesson—one or more periods of learning. They guide lesson design and formative assessment processes. Learning Goals are derived from the Building Blocks of the content standards and the practice standards, and they should state clearly what students will understand or be able to do by the end of the lesson. Teachers might write one or multiple Learning Goals from a Building Block, depending on the depth and scope of the learning it describes.

Success Criteria are derived from Learning Goals, but they are more specific. They explicitly describe student performances of understanding or skills—what students will say, do, make, or write—to demonstrate that they have met the Learning Goals.

## HOW DO LEARNING GOALS AND SUCCESS CRITERIA FIT INTO FORMATIVE ASSESSMENT?

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Formative assessment is a planned process and includes the following:

- (1) Creating learning/teaching progressions between and within standards;
- (2) Establishing clear Learning Goals for the lesson and associated Success Criteria (what students will say, do, make or write to indicate that they have met the goal);
- (3) Sharing Learning Goals and Success Criteria with students and making sure they understand what goals and criteria entail;
- (4) Planning strategies to elicit evidence of learning during the lesson (what students will say, do, make or write);  
*(Note: any Evidence Gathering Strategies need to be aligned to the Learning Goal and Success Criteria. Quality instructional tasks, designed to build students' thinking, can reveal substantive insights into how their thinking is developing - in effect, the instructional task and the assessment task are one and the same)*
- (5) Interpreting the evidence in real-time or as close to the actual time of the lesson as possible to make judgments about where students are in relation to the lesson Learning Goals;
- (6) Deciding on appropriate pedagogical action to move students' learning closer to the desired goal. Possible pedagogical actions include: continuing with the planned lesson; providing feedback that gives students hints or cues about steps they can take; or purposefully departing from the planned lesson in response to the current levels of students' understanding;
- (7) Involving students in the process through peer and self-assessment.

# COOPERATION, COLLEAGUES, AND OUTSIDE RESOURCES

*Standards provide a common set of stated reference points for teacher use, and acquire meaning through use over time. This is because standards, written as verbal descriptors, require interpretation and application within a community of practice (Klenowski & Wyatt-Smith, 2014, p. 77).*

As they plan instruction and formative assessment, teachers are **strongly encouraged** to work in a community of practice, for example with their grade-level peers, Special Education and English Language Development specialists, or other content experts. Insights from professionals who work with special student populations are critical to ensure that appropriate scaffolding and supports for these students are embedded in instruction and formative assessment. The more expertise and perspectives brought to the table, the richer the analysis and formative assessment planning will be, and the more teachers will be able to talk through tricky conceptual questions that may arise.

Effectively creating Building Blocks (and from them, Learning Goals and Success Criteria) will ultimately depend on teachers' deep knowledge of the standards and of their students. The process described in the CSAI resource, *What's Learned First, What's Learned Together? Developing a Yearlong Plan from the K-8 College and Career Ready Standards for Mathematical Content*, can help teachers develop this depth of understanding, which, when combined with their professional knowledge, can provide the necessary foundation for lesson-sizing the standards.

How teachers decide to best parse student learning of particular standards may not be identical to the work others have done in this regard; it will be important to use external resources (see below, as well as the "Additional Resources" section) as points of reference and/or to verify the mathematical soundness of their decisions.

- **Mathematics Progressions from the Institute for Mathematics and Education: University of Arizona.** These narrative documents are organized by domain and describe the progression of a topic across a number of grade levels for the CCSSM, informed by both research of children's cognitive development and the logical structure of mathematics. Information on math progressions helps teachers create Building Blocks that allow for students' growth from less sophisticated to more sophisticated understandings.
- **Learning Trajectories: North Carolina State University, College of Education.** These interactive web applications present "Learning Trajectories" for the CCSSM. These trajectories describe how concepts and student understanding develop over time and can help teachers create Building Blocks that allow for students' growth from less sophisticated to more sophisticated understandings.
- **Mathematics Modules from the New York Department of Education.** The CCSSM for each grade have been clustered into instructional modules that are presented in a yearlong sequence. Each instructional module includes instructional plans that "chunk" learning into lesson-sized portions; these lessons may serve as inspiration for teachers developing their own Learning Goals and Success Criteria.
- **Sample Exercises from Khan Academy.** Exercises aligned to the CCSSM show teachers what the standards look like when translated into problems/activities/questions. Note that a single exercise (or even a collection of exercises) should not be understood to adequately cover a standard or fully address the complexity of learning the standard describes. The exercises might, however, serve as a useful "inspirational device" as teachers create Building Blocks. Reviewing standards-aligned exercises can help remind teachers of some of the "tricky turns" involved in achieving the standard.

# FROM STANDARDS TO BUILDING BLOCKS

The work of articulating a series of Building Blocks is done standard by standard. That is, teachers select a single standard and engage in the following three steps. For each standard, we recommend that teachers allot approximately 45 minutes or an hour. Ideally, teachers will do this work over a longer session (e.g., a full day), so that they can articulate Building Blocks for several related standards in one sitting. The key concepts in related standards are similar, so if these ideas are fresh in teachers' minds, it will be easier to make connections, recognize prior knowledge, and more precisely differentiate between related but distinct standards and Building Blocks.

## Step 1 Identifying the boundaries of prior knowledge.

To determine relevant prior knowledge, teachers begin by consulting the previous grade's standards. Sometimes there are identical domain and cluster headings in the previous grade, so it is easy to identify the relevant prior knowledge described in the standard(s). In other cases, a domain or cluster is new in the current grade, and teachers will need to scrutinize the previous grade's standards more carefully to determine the conceptual roots of the new standard. Teachers who have used the *CCRS Gazette*<sup>4</sup> to create a yearlong plan will be familiar with this grade-to-grade comparison of the standards.

Sometimes the standards *within* a grade build on each other (i.e., there are *within-grade* pre-requisite standards). For example, students in Grade 3 need to "understand concepts of area measurement" (CCSSM.3.MD.5) before they "relate area to the operations of multiplication and addition" (CCSSM.3.MD.7). In cases like these, the prior knowledge for one standard is another, closely related standard *from the same grade*.

There may be prior knowledge that will be critical in the learning of the new standard *that is not expressed in the current or prior grade's standards*. For example, in the CCSSM for Grade 7, students "understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative," and students represent this concept using a number line (7.NS.1b). Teachers familiar with this content will recognize that a key piece of prior knowledge is the ability to use a number line strategically (i.e., scale a number line in a useful way for a given problem, figure out which part of the number line to draw and use in a given problem). This "hidden" prior knowledge is not mentioned in the Grade 6 or Grade 7 standards; awareness of it comes from teachers' experience and content expertise.

## Step 2 Extending the boundaries of prior knowledge.

Before introducing new Building Blocks of learning, there may be useful and productive ways to push students to the edge of their prior learning. These Building Blocks that extend prior knowledge might, for example, involve increasing the *size* of the numbers in a given type of problem that students have solved previously, the *number* of steps in a given process that they are already familiar with, and the *degree* of required accuracy or precision, etc.

For example, new learning in Grade 3 of the CCSSM asks students to "interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each" (3.OA.1). Relevant prior knowledge (2.OA.4) involves being able to "use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends." Before being introduced

<sup>4</sup> The CCRS Gazette is presented in the CSAI resource, *What's Learned First, What's Learned Together? Developing a Yearlong Plan from the K-8 College and Career Ready Standards for Mathematical Content*. It helps teachers analyze, organize, and sequence grade-level standards into a yearlong plan. Visit [csai-online.org](http://csai-online.org) to download this resource.



to the concept of multiplication, Grade 3 students can push the boundaries of their Grade 2 knowledge by working with rectangular arrays with *more* than 5 rows and 5 columns.

### Step 3 Moving beyond the boundaries of prior knowledge to new knowledge.

Here, teachers determine the new incremental learning steps that students will need to experience as they move beyond their prior knowledge. A Building Block that moves student understanding beyond the boundaries of prior knowledge to new knowledge might involve:

- The introduction of a new concept or procedure;
- A new representation of an existing concept or procedure (including new language, symbolic representation, notation, etc.);
- A new application of an existing concept or procedure; or
- A new synthesis of previous learning.

## WHAT DO STEPS 1 – 3 ACTUALLY LOOK LIKE?

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The process of determining the Building Blocks that connect prior knowledge to a new standard should be undertaken collaboratively, with other grade-level teachers or content experts. This conceptual work is iterative: teachers most likely will review and refine the Building Blocks that they've initially developed several times. Teachers should *not* expect to simply list the Building Blocks of a standard, in a suitable order, on the first try. The process described below will likely contain comments like, "Oh wait, we forgot about X... students are going to need to know that before they get to Y," or "Hold on, what do they know about Z from last year?"

For this reason, teachers are encouraged to write and arrange Building Blocks on post-it notes or in a word processing program, instead of trying to list them on a single sheet of paper. A flexible format allows teachers to insert, delete, and reorder Building Blocks throughout the process. After the Building Blocks of a given standard have been determined, this work is recorded and preserved on the "Building Blocks of a Standard" template (for a copy, see the "Templates" section at the end of this resource).

Really getting inside a standard to locate its implicit cognitive moves will likely generate valuable insights from teachers. Dissecting a standard into Building Blocks helps teachers think about:

- Instructional Strategies / Lesson Design;
- Scaffolds/Supports (consider all students, including diverse learners and those in special populations);
- Connections Across Content / Subjects;
- Connections to Real-World Contexts;
- Anticipated Challenges (consider all students, including diverse learners and those in special populations)
- Shifts in Language (from prior to new learning); and
- Math Practice Standards (it is particularly important to note opportunities to merge content and practice standards; these mergers are central to the development of Learning Goals and Success Criteria, described in the next section).

In order not to lose these valuable insights, teachers should jot down such notes as they write each Building Block. These insights allow teachers to think about learning for all students, including diverse learners and those in special populations. In particular, Special Education teachers can record IEP goals in the Building Block notes. For an example of this note taking, see the section below, "Step 2, Illustrated."

Different groups of teachers might make different stylistic choices in the writing of their Building Blocks. Some teachers might prefer to write complete sentences describing each Building Block. Other teachers might prefer to include example exercises, drawings, or keywords. Different grades and different content might be expressed more easily in different formats. What's important is that the series of Building Blocks hits on all the key incremental learning steps between prior knowledge and new knowledge.

This process is illustrated below with an example from the Grade 7 CCSSM. Different groups of teachers may write slightly different Building Blocks, so the example below should be considered as a set of Building Blocks, rather than *the* set of Building Blocks for this standard.<sup>5</sup>

## STEP 1 - IDENTIFYING THE BOUNDARIES OF PRIOR KNOWLEDGE (ILLUSTRATED)

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From a set of grade-level CCRS, teachers select a "target standard."

### TARGET STANDARD

**7.NS.1b** Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Teachers search the prior and current grade's standards and locate any that describe relevant prior knowledge. The guiding question here is, "What do my students already know that they can build on as they move towards the target standard?"

### RELEVANT PRIOR KNOWLEDGE

**6.NS.7a** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*

**6.NS.6a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite.

**6.NS.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.*

Based on their content knowledge and teaching experience, teachers consider whether there is any "hidden" prior knowledge that students will need to begin working towards the "target standard."

<sup>5</sup> Additional examples of Building Blocks for the Grade 3 and Grade 7 CCSSM are included in the "Tools and Exemplars" section. Like this in-text example, these should be considered as typical Building Blocks, not the only "correct" Building Blocks.

**"Hidden" prior knowledge:** Strategically use number line with fluency and precision.

After completing Step 1, teachers have the beginning and end of their pathway of Building Blocks, and they are now ready to determine the intermediate cognitive moves. Note that as teachers articulate these intermediate steps, they may discover more "hidden" prior knowledge. When this happens, it should be added to the list of relevant prior knowledge.

## STEP 2 - EXTENDING THE BOUNDARIES OF PRIOR KNOWLEDGE (ILLUSTRATED)

Teachers consider the prior learning recorded in Step 1, and determine if there are ways to stretch this learning strategically towards the target standard. The guiding question here is, "Before introducing new learning, how can students build strategically on this prior learning?"

BUILDING BLOCKS OF A STANDARD	NOTES (INCLUDE MATH PRACTICE STANDARDS)
<p><b>Block 1 (Extend Prior Knowledge)</b> Understand directionality and vocabulary of number line: "greater" refers to values to the right and "less" refers to values to the left.</p>	<p>(Anticipated Challenge) Students may confuse magnitude and position: "<math>7 &gt; 3</math>" is intuitive, but "<math>-7 &lt; 3</math>" may be confusing.</p> <p>(Instructional Strategy) For ELs and others who need language support, use visual aids to introduce and consistently reinforce this concept and vocabulary.</p>

## STEP 3 - MOVING BEYOND THE BOUNDARIES OF PRIOR KNOWLEDGE (ILLUSTRATED)

Here, teachers do the bulk of the work of the Building Block process. Starting from the last Building Block from Step 2, they determine the incremental steps along the conceptual pathway to the target standard. The guiding question here is, "Given that I know X, what is the next thing I need to learn to move towards the target standard?"

BUILDING BLOCKS OF A STANDARD	NOTES (INCLUDE MATH PRACTICE STANDARDS)
<p><b>Block 2 (Move beyond prior knowledge)</b> Reinterpret simple addition problems of the form, "<math>p + q</math>," (single digits, positive second addend) as movement on a number line. EX: <math>1+2</math>, <math>4+3</math>, <math>-3+3</math>, <math>-1+2</math>, <math>-4+3</math></p>	<p>(Instructional Strategy) Use consistent language: first addend is "where you start," second addend is "how far you move."</p> <p>(Connections Across Subjects) Relate this to temperature change, elevation change, bank account balances.</p> <p>(Math Practice Standard 1) Compare answers from two methods, arithmetic and number line.</p> <p>(Anticipated Challenge) Students may have memorized rules signed addition (e.g., "when you add a negative and a positive, subtract and then use the sign of the large number") and may be hesitant to move to a number-line-based understanding.</p>
<p><b>Block 3 (Move beyond prior knowledge)</b> Reinterpret addition problems of the form, "<math>p + (-q)</math>," (single digits, negative second addend) as movement on a number line. EX: <math>1+(-2)</math>, <math>4+(-3)</math>, <math>3+(-3)</math>, <math>-1+(-2)</math>, <math>-4+(-3)</math></p>	<p>See notes from Block 2.</p> <p>(Anticipated Challenge) For all students, and ELs in particular, there may be confusion about the vocabulary and concepts of "minus" and "negative." Consistently use the form "<math>p + q</math>" and parentheses for the second addend.</p> <p>(Math Practice Standard 3) Students can now explain why <math>3 + (-3) = 0</math> in terms of movement on a number line.</p>
<p><b>Block 4 (Move beyond prior knowledge)</b> Recognize the repeated reasoning in Blocks 3 and 4. EX: compare <math>1+2</math> &amp; <math>1+(-2)</math>, <math>4+3</math> &amp; <math>4+(-3)</math>, <math>3+3</math> &amp; <math>3+(-3)</math>, <math>-1+2</math> &amp; <math>-1+(-2)</math>, <math>-4+3</math> &amp; <math>-4+(-3)</math></p>	<p>(Math Practice Standard 7) This Block is about understanding and using the general structure, "<math>p + q</math>," in terms of absolute value and number line movement.</p> <p>(Instructional Strategy) Students can synthesize this learning on posters or anchor charts, which will make it easier to build on this learning for standard 7.NS.1c.</p>

## RECORDING THE WORK OF STEPS 1 – 3 IN THE "BUILDING BLOCKS OF A STANDARD" TEMPLATE

Teachers who have completed Steps 1 - 3 now have a series of Building Blocks that trace the learning pathway from the prior grade's standard(s) to the target standard. If this analysis was done on post-it notes or in a word processing program, it can now be transferred to the "Building Blocks of a Standard" template. This template will be used as a key reference in developing Learning Goals and Success Criteria, the topic of the following section. A completed template (based on the Grade 7 example developed throughout this section) is included below. A Grade 3 example of the "Building Blocks of a Standard" template is included in the "Tools and Exemplars" section.

### TARGET STANDARD

**7.NS.1b** Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

### RELEVANT PRIOR KNOWLEDGE

**6.NS.7a** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*

**6.NS.6a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite.

**6.NS.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.*

**"Hidden" prior knowledge:** Strategically use number line with fluency and precision.

BUILDING BLOCKS OF A STANDARD	NOTES (INCLUDE MATH PRACTICE STANDARDS)
<p><b>Block 1 (Extend prior knowledge)</b> Understand directionality and vocabulary of number line: "greater" refers to values to the right and "less" refers to values to the left.</p>	<p>(Anticipated Challenge) Students may confuse magnitude and position: "<math>7 &gt; 3</math>" is intuitive, but "<math>-7 &lt; 3</math>" may be confusing.</p> <p>(Instructional Strategy) For ELs and others who need language support, use visual aids to introduce and consistently reinforce this concept and vocabulary.</p>
<p><b>Block 2 (Move beyond prior knowledge)</b> Reinterpret simple addition problems of the form, "<math>p + q</math>," (single digits, positive second addend) as movement on a number line. EX: <math>1+2</math>, <math>4+3</math>, <math>-3+3</math>, <math>-1+2</math>, <math>-4+3</math></p>	<p>(Instructional Strategy) Use consistent language: first addend is "where you start," second addend is "how far you move."</p> <p>(Connections Across Subjects) Relate this to temperature change, elevation change, bank account balances.</p> <p>(Math Practice Standard 1) Compare answers from two methods, arithmetic and number line.</p> <p>(Anticipated Challenge) Students may have memorized rules signed addition (e.g., "when you add a negative and a positive, subtract and then use the sign of the large number") and may be hesitant to move to a number-line-based understanding.</p>
<p><b>Block 3 (Move beyond prior knowledge)</b> Reinterpret addition problems of the form, "<math>p + (-q)</math>," (single digits, negative second addend) as movement on a number line. EX: <math>1+(-2)</math>, <math>4+(-3)</math>, <math>3+(-3)</math>, <math>-1+(-2)</math>, <math>-4+(-3)</math></p>	<p>See notes from Block 2.</p> <p>(Anticipated Challenge) For all students, and ELs in particular, there may be confusion about the vocabulary and concepts of "minus" and "negative." Consistently use the form "<math>p + q</math>" and parentheses for the second addend.</p> <p>(Math Practice Standard 3) Students can now explain why <math>3 + (-3) = 0</math> in terms of movement on a number line.</p>
<p><b>Block 4 (Move beyond prior knowledge)</b> Recognize the repeated reasoning in Blocks 3 and 4. EX: compare <math>1+2</math> &amp; <math>1+(-2)</math>, <math>4+3</math> &amp; <math>4+(-3)</math>, <math>3+3</math> &amp; <math>3+(-3)</math>, <math>-1+2</math> &amp; <math>-1+(-2)</math>, <math>-4+3</math> &amp; <math>-4+(-3)</math></p>	<p>(Math Practice Standard 7) This Block is about understanding and using the general structure, "<math>p + q</math>," in terms of absolute value and number line movement.</p> <p>(Instructional Strategy) Students can synthesize this learning on posters or anchor charts, which will make it easier to build on this learning for standard 7.NS.1c.</p>

# FROM BUILDING BLOCKS TO LEARNING GOALS AND SUCCESS CRITERIA

Once teachers have determined the Building Blocks that connect one standard to another, they are ready to use these Building Blocks to develop Learning Goals and Success Criteria for individual lessons. The work of this section will be recorded in the "Lesson-Sized Learning Goals and Success Criteria" template (found at the end of this resource). To do this work, teachers will need a filled-in "Building Blocks of a Standard" template. Additionally, because Math Practice Standards will be incorporated into Learning Goals and Success Criteria, teachers who teach from such standards will need a copy of them.

## Step 1

### **Stacking building Blocks: Determining how many building blocks belong in a lesson.**

Remember that Building Blocks are usually lesson-sized, meaning that one Building Block is taught per lesson. Occasionally, Building Blocks may describe much smaller learning steps, in which case it will make sense from an instructional perspective to address multiple Building Blocks in a single lesson. The first step, then, in developing the Learning Goals and Success Criteria of a lesson, is to decide how many Building Blocks ought to be stacked together in a lesson. This decision is made based on teachers' content knowledge, teaching experience, and familiarity with their students.

For example, based on the four Grade 7 Building Blocks developed in the previous section, a group of teachers decided that the first two stack together well instructionally. That is, the learning of the first Block is very closely related to the learning of the second Block, the two Blocks combined describe a lesson-sized amount of learning, and it will not be cognitively overwhelming for students to engage with both Blocks simultaneously. The third and fourth Building Blocks can also be stacked together in a lesson.

Note that some Building Blocks associated with procedural skills and fluency should not be stacked with any other Building Blocks. This is because they will need to be included repeatedly in several lessons to allow students adequate practice.

## Step 2

### **Developing learning goals and success criteria.**

In the development of Learning Goals and Success Criteria, teachers can very meaningfully merge mathematical content and practice standards. *How* students will engage with mathematics (i.e., practice standards) should be reflected just as much as *what* mathematics they will learn (i.e., content standards).

Teachers should refer to the "Building Blocks of a Standard" template and review any notes they have written for the lesson's Building Block(s). The following table provides guidance in writing Learning Goals and Success Criteria and helps teachers differentiate between the two. An expanded version of this table (one that includes math practice standards) is reproduced in the "Tools and Exemplars" section.)

## Learning Goals, Success Criteria, and Practice Standards

Created by CRESST, UCLA, 2015

Learning Goals	Success Criteria
Describe what learning students are to develop (an understanding, principled knowledge, skill, or a process) as a result of this lesson.	Describe what students need to say, do, make, or write to show that they have met the goal (i.e., what is the performances of a skill, understanding, etc.?).
Start with a verb (e.g., develop, become fluent, apply, understand). The Practice Standards contain many such verbs and are a valuable resource in articulating and phrasing Learning Goals.	Start with a verb (e.g., explain, describe, model). The Practice Standards contain many such verbs and are a valuable resource in articulating and phrasing Success Criteria.
Be sure that the learning goal is manageable within the context of one lesson.	Be sure that the Success Criteria are aligned with the Learning Goal, and are indications of achievement of the Goal.
Write in language that is understandable to students.	Write in language that is understandable to students so they can use the criteria to monitor their own learning. Teachers will need to explain the Success Criteria at the outset of a lesson, and provide exemplars if necessary, to make sure students understand what is expected of them.

Generally, a stack of Building Blocks should have one or two (usually no more than three) Learning Goals. They should be appropriately complex and comprehensive enough to address the learning reflected in the lesson's Building Blocks. Each Learning Goal may have several associated Success Criteria. A quick rule of thumb is, "fewer Learning Goals, more Success Criteria."

### Step 3 Ensuring a merger between the content and practice standards.

The final step, after developing a lesson's Learning Goals and Success Criteria, is to make sure this work meaningfully integrates content and practice standards. Teachers review their completed "Lesson-Sized Learning Goals and Success Criteria" template and annotate places where math practice standards are reflected in student learning. If this annotation does not include several instances, teachers return to Step 2 to modify their Learning Goals and Success Criteria or add more, ensuring that the practice standards are thoroughly integrated into the teaching and learning of the lesson.

## RECORDING THE WORK OF STEPS 1 – 3 IN THE "LESSON-SIZED LEARNING GOALS AND SUCCESS CRITERIA" TEMPLATE

The Learning Goals and Success Criteria for each stack of Building Blocks can be recorded on an individual template, or all of the stacks for a given standard can be recorded on the same template. Two completed "Lesson-Sized Learning Goals and Success Criteria" templates (based on the Grade 7 example developed throughout this section) are included below. A Grade 3 example of this template is included in the "Tools and Exemplars" section.

# Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

Name: Ms. Branch Grade: 7 Year: 2014-2015

Subject/Period: math

Standard: CCSS.MATH.CONTENT.7.NS.A.1.B

Understand  $p + a$  as the number located a distance  $|a|$  from  $p$ , in the positive or negative direction depending on whether  $a$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)
<p>Block 1</p> <p>Understand directionality and vocabulary of number line: "greater" refers to values to the right and "less" refers to values to the left.</p>	<p>Differentiate between the concepts of magnitude and position for signed numbers.</p>	<p>Correctly select "&lt;" or "&gt;" in inequalities of the following type  <math>-3 \_ -7</math>, <math>3 \_ -7</math>, <math>-7 \_ -3</math> and then justify their selection.</p> <p>Clearly explain why <math>-7</math> is less than <math>3</math>, even though <math>7</math> is a larger number than three.</p>
<p>Block 2</p> <p>Reinterpret simple addition problems of the form, "<math>p + a</math>," (single digits, positive second addend) as movement on a number line.</p> <p>Ex: <math>1+2</math>, <math>4+3</math>, <math>-3+3</math>, <math>-1+2</math>, <math>-4+3</math></p>	<p>Make strategic use of the number line to represent signed addition with a positive second addend (e.g., <math>p + a</math>, where <math>a</math> is positive).</p> <p>Understand that signed addition with a positive second addend (e.g., <math>p + a</math>, where <math>a</math> is positive) can be used to solve real-world problems (e.g., temperature change, elevation change, bank balances, etc.).</p>	<p>Correctly represent problems of the type, "<math>p + a</math>, where <math>a</math> is positive" using arrows on a number line.</p> <p>Accurately write this type of addition problem from an illustrated number line diagram with arrows.</p> <p>Clearly and correctly explain the role of the first and second addend in number line addition.</p> <p>Correctly describe a real-world scenario that could be modeled from a given addition problem of this type.</p> <p>Correctly model a given real-world scenario with an addition problem of this type.</p>

## Continued: Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)
<p>Block 3</p> <p>Reinterpret addition problems of the form, "<math>p + (-a)</math>," (single digits, negative second addend) as movement on a number line.</p> <p>Ex: <math>1+(-2)</math>, <math>4+(-3)</math>, <math>3+(-3)</math>, <math>-1+(-2)</math>, <math>-4+(-3)</math></p>	<p>make strategic use of the number line to represent signed addition with a negative second addend (e.g., <math>p + a</math>, where <math>a</math> is negative).</p> <p>understand that signed addition with a negative second addend (e.g., <math>p + a</math>, where <math>a</math> is negative) can be used to solve real-world problems (e.g., temperature change, elevation change, bank balances, etc.).</p>	<p>correctly represent problems of the type, "<math>p + a</math>, where <math>a</math> is negative" using arrows on a number line.</p> <p>accurately write this type of addition problem from an illustrated number line diagram with arrows.</p> <p>use the number line to justify and clearly explain why <math>p + (-p) = 0</math>.</p> <p>clearly and correctly explain the role of the first and second addend in number line addition.</p> <p>correctly describe a real-world scenario that could be modeled from a given addition problem of this type.</p> <p>correctly model a given real-world scenario with an addition problem of this type.</p>
<p>Block 4</p> <p>Recognize the repeated reasoning in Blocks 3 and 4.</p> <p>Ex: compare <math>1+2</math> &amp; <math>1+(-2)</math>, <math>4+3</math> &amp; <math>4+(-3)</math>, <math>3+3</math> &amp; <math>3+(-3)</math>, <math>-1+2</math> &amp; <math>-1+(-2)</math>, <math>-4+3</math> &amp; <math>-4+(-3)</math></p>	<p>understand that "<math>p + a</math>" describes a movement of <math>a</math> units from <math>p</math> on a number line.</p> <p>appreciate the necessity of the absolute value sign (for problems where <math>a</math> is negative).</p>	<p>clearly and correctly explain when <math>p + a</math> describes movement in the positive direction and when it describes movement in the negative direction.</p> <p>justify and clearly explain the use of absolute value in the statement "<math>p + a</math> means start at <math>p</math> and move <math>a</math> units."</p>



# TOOLS AND EXEMPLARS

## GUIDES

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### LEARNING GOALS, SUCCESS CRITERIA, AND PRACTICE STANDARDS

This quick guide can be used as a reference for developing Learning Goals and Success Criteria that integrate standards for mathematical practice. The math practice standards in this guide are from the CCSSM. Teachers who teach from College and Career Readiness Standards other than the CCSS may work with different sets of practice standards.

## COMPLETED EXAMPLES

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### BUILDING BLOCKS OF A STANDARD

This example illustrates how a teacher might articulate the Building Blocks of standard 3.OA.1 of the Grade 3 CCSSM.

### LESSON-SIZED LEARNING GOALS AND SUCCESS CRITERIA

This example illustrates how a teacher might articulate Learning Goals and Success Criteria for the Building Blocks articulated in the above example.

# Learning Goals, Success Criteria, and Practice Standards

Created by CRESST, UCLA, 2014

Learning Goals	Success Criteria
Describe what learning students are to develop (an understanding, principled knowledge, skill, or a process) as a result of this lesson.	Describe what students need to say, do, make, or write to show that they have met the goal (i.e., what is the performances of a skill, understanding, etc.?).
Start with a verb (e.g., develop, become fluent, apply, understand). The Practice Standards contain many such verbs and are a valuable resource in articulating and phrasing Learning Goals.	Start with a verb (e.g., explain, describe, model). The Practice Standards contain many such verbs and are a valuable resource in articulating and phrasing Success Criteria.
Be sure that the learning goal is manageable within the context of one lesson.	Be sure that the Success Criteria are aligned with the Learning Goal, and are indications of achievement of the Goal.
Write in language that is understandable to students.	Write in language that is understandable to students so they can use the criteria to monitor their own learning. Teachers will need to explain the Success Criteria at the outset of a lesson, and provide exemplars if necessary, to make sure students understand what is expected of them.

## Standards for Mathematical Practice

- (1) Make sense of problems and persevere in solving them
- (2) Reason abstractly and quantitatively
- (3) Construct viable arguments and critique the reasoning of others
- (4) Model with mathematics
- (5) Use appropriate tools strategically
- (6) Attend to precision
- (7) Look for and make use of structure
- (8) Look for and express regularity in repeated reasoning

# Building Blocks of a Standard

Created by CRESST, UCLA, 2014

Name: MS. Fields

Grade: 3

Year: 2014-2015

## TARGET STANDARD

3.OA.1 Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .

## RELEVANT PRIOR KNOWLEDGE

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.  
 "Hidden" prior knowledge: Arrange objects in groups and arrays with fluency and precision.

BUILDING BLOCKS OF A STANDARD	NOTES (INCLUDE MATH PRACTICE STANDARDS)
Block 1 - Practice repeated addition of objects arranged in rectangular arrays with progressively more rows and columns (beyond 5 rows and 5 columns). EX $7+7+7+7$ and $2+2+2+2+2+2+2$	(Instructional Strategy) Students should practice this until it is no longer cognitively demanding so that they are receptive to the idea of multiplication as a "shortcut to repeated addition."
Block 2 - move between symbolic ( $2+2+2$ ) and concrete (four groups of 2 objects) representations of the same repeated addition number sentence.	(math practice standard 2) Students need to be comfortable toggling between symbolic and concrete representations of repeated addition.  (Connections Across Subjects) They can practice using these different representations in science (planning rows of plants for a class garden, counting tally marks in data collection) and art (counting, grouping, and arranging shapes, objects, colors).
Block 3 - Describe repeated addition like $2+2+2$ as "the number 2, added four times," and then, "four times 2."	(Shift in Language) This important linguistic Building Block prepares students for the conceptual shift from addition to multiplication.  (math practice standard 6) It will be important for all students, and ELs in particular, to use this new language with precision, as it is deeply connected to the underlying conceptual shift from addition to multiplication
Block 4 - Extend the learning of Block 3 to include more repeats. EX $2+2+2+2+2+2$	See notes from Block 3.
Block 5 - Extend the learning of Block 3 to include more objects in each group. EX $7+7+7+7$	See notes from Block 3.
Block 6 - Understand multiplication as a shortcut to repeated addition.	(Instructional Strategy) As appropriate, encourage students to use the 100's square, addition table, and multiplication table.  (math practice standard 8) Students can begin to notice regularities in the 100's square, addition table, and multiplication table, and express these regularities in terms of multiplication.

# Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

Name: MS. Fields Grade: 3 Year: 2014-2015

Subject/Period: math

Standard: CCSS.MATH.CONTENT.3.OA.A

Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)
<p>Block 1 Practice repeated addition of objects arranged in rectangular arrays with progressively more rows and columns (beyond 5 rows and 5 columns).</p> <p>EX <math>7+7+7+7</math> and <math>2+2+2+2+2+2</math></p>	<p>Become fluent in computing repeated addition with more than 5 repeats and more than 5 grouped objects.</p>	<p>Solve repeated addition problems (from symbolic and concrete representations) with precision, accuracy, and speed.</p>
<p>Block 2 Move between symbolic (<math>2+2+2</math>) and concrete (four groups of 2 objects) representations of the same repeated addition number sentence.</p>	<p>Understand that repeated addition can be represented with a number sentence or with a concrete representation (e.g., manipulatives arranged in an array).</p>	<p>Write a correct number sentence from a given concrete representation of repeated addition.</p> <p>Accurately explain the correspondence between their number sentence and the given representation.</p> <p>Correctly model a given number sentence about repeated addition with a concrete representation.</p> <p>Accurately explain the correspondence between their concrete representation and the given number sentence.</p>
<p>Block 3 Describe repeated addition like <math>2+2+2</math> as "the number 2, added four times," and then, "four times 2."</p>	<p>Recognize the structure of repeated addition and understand that repeated addition can be expressed as the number of times a number repeats.</p>	<p>Create accurate number sentences using repeated addition, from a given set of objects.</p> <p>Make an accurate drawing of their number sentence.</p> <p>Correctly describe the number of repeats in a concrete representation (e.g., "I have 6 repeats of this set of 3 things").</p> <p>Correctly and precisely use the vocabulary "times" to express the number of repeats.</p>

# Continued: Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)
<p>Block 4 Extend the learning of Block 3 to include more repeats.</p> <p>EX <math>2+2+2+2+2+2</math></p> <p>Block 5 Extend the learning of Block 3 to include more objects in each group.</p> <p>EX <math>7+7+7+7</math></p>	<p>Extend their understanding of multiplication as repeated addition to (1) larger sets and (2) more repeats.</p>	<p>Write a correct number sentence from a given concrete representation of repeated addition (with larger sets and more repeats).</p> <p>Accurately explain the correspondence between their number sentence and the given representation.</p> <p>Correctly model a given number sentence about repeated addition with a concrete representation (with larger sets and more repeats).</p> <p>Accurately explain the correspondence between their concrete representation and the given number sentence.</p>
<p>Block 6 Understand multiplication as a shortcut to repeated addition.</p>	<p>Understand that the symbol, "x" can be used as a shortcut to repeated addition.</p>	<p>Accurately explain the meaning of the two terms in the "x" number sentence.</p> <p>Write a correct number sentence with "x" from a given number sentence with "+."</p> <p>Correctly describe where the two terms of the "x" number sentence can be seen in the "+" number sentence.</p> <p>Write a correct number sentence with "+" from a given number sentence with "x."</p> <p>Adequately justify their choice of addend and their choice of repeats, referencing the "x" number sentence.</p> <p>Correctly describe where the addends and repeats of the "+" number sentence can be seen in the "x" number sentence.</p>

# ADDITIONAL RESOURCES

The following resources contain a variety of information that supports math teachers in analyzing CCRS and planning for formative assessment.

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## **EQuIP RUBRICS AND EXEMPLAR LESSONS FROM ACHIEVE**

<http://www.achieve.org/EQuIP>

Achieve is a nonprofit education reform organization that works with states around academic standards, graduation requirements, assessments, and accountability. The organization has played a central role in the development of the Common Core State Standards and the Next Generation Science Standards. By following the link above, teachers can access a variety of free educational materials, including:

- "EQuIP" (Educators Evaluating the Quality of Instructional Products) rubrics that are used to evaluate the quality of existing instructional resources, and
  - "EQuIP" exemplar lessons, which have been carefully vetted for quality and alignment to the cognitive demands of the CCSS.
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## **LEARNING TRAJECTORIES: NORTH CAROLINE STATE UNIVERSITY, COLLEGE OF EDUCATION**

<http://www.turnonccmath.net/>

These interactive web applications present "Learning Trajectories" for the CCSSM. These trajectories analyze the K-8 standards from the standpoint of student learning and elaborate the underlying scientific research in mathematics education. The website is designed to support teachers, teacher educators, professional development providers, and district- and state-level curriculum experts in interpreting the CCSSM for instructional implementation.

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## **MATHEMATICS PROGRESSIONS FROM THE INSTITUTE FOR MATHEMATICS AND EDUCATION: UNIVERSITY OF ARIZONA**

<http://ime.math.arizona.edu/progressions/>

The Institute for Mathematics and Education supports local, national, and international projects in mathematics education that focus on both mathematics and students, which can be applied to current needs, build on existing knowledge, and are grounded in the work of educators. The Institute is currently involved in

- organizing and writing progression documents for the K–12 Common Core State Standards in Mathematics; these progressions can be found on the Institute's website; and
- elaborating and interpreting the math practice standards for grades K-5 and 6-8; these documents can be accessed through Bill McCallum's blog (see "Tools for the Common Core Standards" below).

## **MATHEMATICS MODULES FROM THE NEW YORK DEPARTMENT OF EDUCATION**

<http://www.engageny.org/mathematics>

The New York State Department of Education has developed Mathematics modules for grades K-12 that are available to educators on their website. These modules focus on reasoning, practice, and reflection, and set high expectations for mastery. The modules include topic overviews, daily lesson plans, extensive problem sets, guiding questions, and examples of proficient student work.

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## **SAMPLE EXERCISES FROM KHAN ACADEMY**

[www.khanacademy.org/commoncore](http://www.khanacademy.org/commoncore)

Exercises aligned to the CCSSM show teachers what the standards look like when translated into problems/activities/questions. The exercises focus on conceptual understanding, procedural fluency, and real-world application, and detailed solutions for each exercise are included.

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## **TOOLS FOR THE COMMON CORE STANDARDS**

<http://commoncoretools.me/>

Bill McCallum, one of the lead authors of the Common Core State Standards for Mathematics, manages this blog cooperatively with co-authors from the Illustrative Mathematics Project and the Institute for Mathematics and Education. The blog presents updates and reports on projects that support the implementation of the CCSSM.

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# TEMPLATES

The following templates are provided for teachers to print and fill in as they analyze and parse their grade-level CCRS.

## **"BUILDING BLOCKS OF A STANDARD"**

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This tool is used to organize teachers' analysis of grade-level math CCRS in an instructionally meaningful way.

## **"LESSON-SIZED LEARNING GOALS AND SUCCESS CRITERIA"**

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This tool is used to record the Learning Goals and Success Criteria of lesson-sized stacks of Building Blocks. Teachers may choose to use one template per stack or record multiple stacks on the same template.

# Building Blocks of a Standard

Created by CRESST, UCLA, 2014

Name: \_\_\_\_\_ Grade: \_\_\_\_\_ Year: \_\_\_\_\_

## TARGET STANDARD

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## RELEVANT PRIOR KNOWLEDGE

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BUILDING BLOCKS OF STANDARD	NOTES (INCLUDE MATH PRACTICE STANDARDS)

# Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

Name: \_\_\_\_\_ Grade: \_\_\_\_\_ Year: \_\_\_\_\_

Subject/Period: \_\_\_\_\_

Standard: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)

# Continued: Lesson-Sized Learning Goals and Success Criteria

Created by CRESST, UCLA, 2014

BUILDING BLOCKS OF A LESSON	LEARNING GOALS (INCLUDE MATH PRACTICE STANDARDS)	SUCCESS CRITERIA (INCLUDE MATH PRACTICE STANDARDS)