Project Overview

What are FAM Tasks?

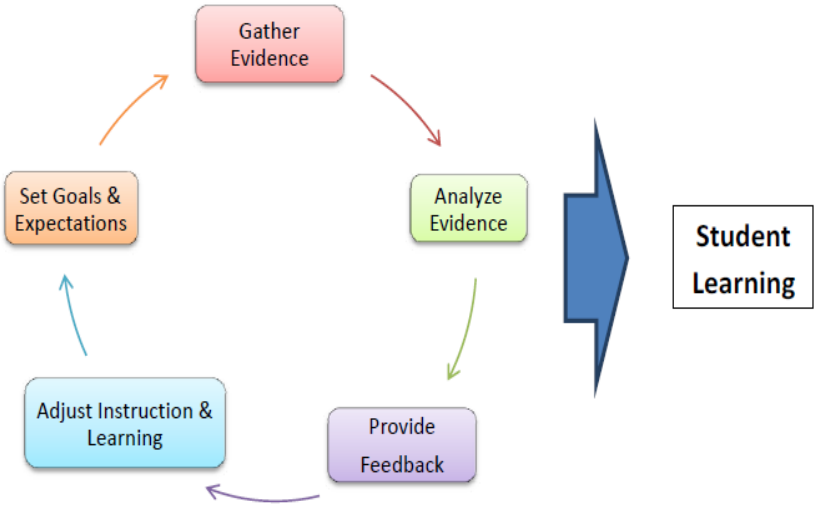
A team of researchers at the National Center for Research on Evaluation, Standards, and Student Testing (CRESST) at UCLA have developed a series of formative math tasks for Grade 8 students with input from content experts. The tasks specifically target Common Core State Standards (CCSS) 8EE5-6 although the overarching structure could be adapted to different grade levels and standards. The structure resembles an exercise routine. Similar to a routine, students are given progressively more challenging tasks to complete. Teachers help "spot" their students in this process via probing questions. Emphasis is placed on students' progress towards a learning goal instead of score on the task. In this process, rich information or evidence is gathered about students' understanding of key concepts. Teachers are also given support in analyzing this evidence, providing feedback to their students and determining instructional next steps.

What is formative assessment?

One of the most widely shared definitions for formative assessment comes from the Council of Chief State School Officers (CSSO). Formative assessment is:

> "*a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes".* (2006)

Formative assessment can take many forms and may involve a range of tools and strategies including informal teacher-student conferences, quizzes or open-ended tasks. The formative assessment process is a cycle, encompassing five broad steps:



FAM tasks are designed to instantiate the formative assessment process or give participants a taste of what it is like to engage in all aspects of the formative assessment cycle.

This work was supported by grant number #DRL-1020393 from the National Science Foundation and grant number 2012-8075 from the William and Flora Hewlett Foundation.

Task Objectives:

By engaging in FAM tasks….

Students will understand that:

- There are many ways to represent patterns/relationships between quantities or units (e.g., graphically, numerically, and symbolically).
- Proportional relationships express how two quantities change relative to each other.
- The same proportional thinking can be applied to many different situations involving rates (e.g., we can create a graph for miles per hour, brownies sold per hour, etc.)
- We encounter proportional relationships and the concept of slope in of our everyday lives.

Students will be able to:

- Determine unit rate using tables, graphs, and equations.
- Connect unit rate with rate of change and slope.
- Describe key concepts such as proportional relationships and slope in their own words.
- Compare/make connections between expressions and graphical representations.
- Identify and interpret slope, equation for slope.
- Utilize similar triangles as way to explain why slope *m* is the same between any two distinct points.
- Practice writing about math, specifically their reasoning related to math tasks.
- Compare their work to teacher-shared expectations and examples to gain a sense of where they are now in relation to where they need to go.

Teachers will:

- Gather diagnostically rich student work.
- Analyze this work for common errors, student understanding of key concepts.
- Use information and insight gained through the analysis of student work to provide feedback to students in relation to learning goals and expectations to determine next steps.

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Common Core Standards and Math Practices Targeted:

While the broad outline of FAM tasks could be used across content standards and/or grade levels, the current tasks focus on two key Common Core State Standards:

> **8EE.5** - Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

> **8EE.6** - Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

The tasks also focus on Math Practice (MP) #3, *Construct viable arguments and critique the reasoning of others.*

Learning Progressions Associated with Target Standards

Understanding mathematics and how to apply mathematics to the real world (e.g., make sense of problems, model with mathematics, look for and make use of structure) requires that students recognize patterns in what they see. Ultimately, a solid conceptual understanding of mathematics requires that students not only recognize, but also can generalize (abstract) properties of numbers and use mathematical operations to describe the situations they encounter in the world surround (e.g. understand and use fractions, understand what multiplication and division means, and understand why the procedures for multiplying and dividing fractions make sense (see 5th grade overview)).

The power of mathematics derives from our ability to use mathematical models to predict the universe surrounding us. We have used mathematical models to place man on the moon, rovers on Mars and to fly by Pluto. We use mathematical models to calculate interest when borrowing money, to calculate the "break-even" point in optimization decisions, and we use mathematical models to construct the environments (buildings, HVAC, utilities, etc.) in which we work and live. Developing such models (MP 4) and using those models to explain patterns (MP 8) are key to the progressions in the math standards.

Before High School, these patterns and models are limited to linear relationships. In grades K - 5, students learn the numbers are ordered (K.CC.4), that **addition** (and subtraction) can be used to **generate** other numbers (K.OA.2), and that these principles can be extended to all real numbers (3.NBT.2, 4.NBT, 5NBT). In the second half of the elementary grades, students are asked to **identify** the patterns they see (3.OA.9), use addition and multiplication to generate numbers from other numbers (i.e., sequences) using mathematical **rules** (4.OA.5) and, finally, to **compare** two sequences, each generated by different rules (5.OA.3). Students make these comparisons both numerically and visually (4.NF.1, 4.NF.4.c, 5.OA.3, 5NF.5), and not only learn to generate numbers using integer multiplication and addition, but also fraction (rational) multiplication (4.NF.4). These activities form an important foundation that will culminate, at the end of middle school, with students

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writing general equations that describe linear relationships between two quantities (independent and dependent variables).

Beginning in 6th grade, students extend their prior understanding to accommodate negative numbers (6.NS.5), and the ratio between two numbers (6.RP.1). Most importantly, students in 6th grade begin to generalize the relationships between two sets of numbers using variables, and to analyze the relationships using graphs and tables (6.EE.9). They will continue to develop their ability to represent the relationship between proportional variables in 7th grade by: a) determining if two variables are in a proportional relationship (7.RP.2.A); b) representing a given [verbally described] proportional relationship by an equation (7.RP.2.C); and, c) using these proportional relationships to solve problems (7.RP.3).

The pinnacle of this progression, prior to high school, comes in grade 8 when students actually "**derive** the equation y = mx" to represent such proportional relationships and the equation y = mx + b to further generalize this relationship to linear but non-proportional associations (8.EE.6). Eighth grade students compare the relationship between two numbers "algebraically, graphically, numerically in tables, or by verbal descriptions" (8.F.2). In high school, this same idea of patterns will again expand to include non-linear equations (e.g., quadratic, rational, exponential – A-CED.1 and F-LE), complex numbers (N-CN), finite geometric series (A-SSE.4), trigonometric functions (F-TF), and statistical relationships (S-ID.7).

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Grade level	Common Core Standards
Kindergarten	Numbers are ordered (K.CC.4.a) Sequences of numbers can be generated from other numbers by repeatedly adding or subtracting the same number within 10 (K.OA)
1st grade	Sequences are expanded to include adding and subtracting numbers within 20 (1.OA.1).
2nd grade	Sequences are expanded to include adding and subtracting numbers within 100 (2.NBT.5)
3rd grade	Sequences are expanded to include multiplying and dividing numbers within 100 (3.OA.3)
4th grade	Distinguish multiplicative from additive sequences (4.OA.2) Generate a pattern [sequence] that follows a rule (4.OA.5) Sequences are expanded to include fractions and decimals (4.NF.1, 4.NF.3, 4.NF.4, 4.NF.6)
5th grade	Generate two numerical patterns using two given rules and identify the relationships between corresponding terms of the two sequences (5.OA.3). Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (5.OA.3) Interpret a sequence generated by rational number multiplication in terms of scaling (5.NF.5)
6th grade	Sequences are expanded to include negative numbers (6.NS.6) Understand and explain the relationship between numbers within one sequence or between two sequences using ratio / rate (6.RP.1) Use equations with variables to represent the relationship of one variable (the dependent variable) to one another variable (the independent variable) (6.EE.9).
7th grade	Recognize and represent two sequences that are proportional in nature (7.RP.2) Use variables and simple equations to represent quantities in a real-world situation (7.EE.4)
8th grade	Compare proportional relationships represented in different ways (e.g., graphs, equations, tables) (8.EE.5) Derive the equation y = mx + b to represent the non-vertical, linear relationship between a dependent (y) and an independent (x) variable (8.EE.6)

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Guidelines for Task Development:

In the course of task development, the FAM team developed a set of guidelines to steer our task development. These guidelines were gathered from numerous sources, particularly from members of the FAM Technical Advisory Committee (TAC). Specifically, we sought to create tasks that are appropriate, complex yet accessible, engaging, feasible and, of course, formative.

Task Development Guidelines	
Appropriate	• Touch upon key progressions within target standards/concepts in logical order or "spiral" (i.e., face validity) (Zimba, 2014). • Are mathematically correct (including the appropriate use of units).
Complex	• Promote conceptual and procedural understanding (Hiebert and Grouws, 2007). • Target and reveal complex thinking in relation to mathematical content and practices (Hiebert and Grouws, 2007).
Accessible	• Utilize language that is accessible for all students. • Provide supports for students with particular needs (e.g., visuals to assist English Learners, scaffolding for students at different levels) (TAC Meeting, 2014). • Grow from simple to complex and provide entry points for students at different levels (Pashler, Rohrer, Cepeda, and Carpenter, 2007). • Tap prior knowledge and make explicit connections among concepts • Use clear and purposeful diagrams, pictures, graphs and text.
Engaging	• Connect to real-life situations and issues. • Promote talking, writing, thinking about math (Smith & Stein, 2011).
Feasible	• Be feasible for a teacher to implement within the given time period in an average Grade 8 math classroom and provide him/her with timely information.
Formative	• Identify clear learning goals and success criteria (e.g., Heritage, 2011; Moss, Brookhart, and Long, 2013). • Provide opportunities for peer and self-assessment (Sadler, 1989). • Provide meaningful feedback to students based on the success criteria (e.g., Black & William, 1998; Hattie & Timperley, 2008). • Allow for students and teachers to adjust learning based on information (differentiation and choice) (Heritage, Kim, Vendlinski, and Herman, 2009). • Gather diagnostically rich information helpful to teachers in analyzing students' understanding and reflecting on instructional next steps (e.g., Black & William, 1998; Heritage, Kim, Vendlinski, and Herman, 2009).

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Structure for Tasks:

Each task is comprised of several steps, similar to an exercise routine. In the course of the routine, students have repeated opportunities to engage in increasingly complex activities linked to target content standards and math practices while teachers gather rich information about students' understanding and provide formative feedback. We have written each task so that a sequence of activities - Set-up, Mind Stretch, Workout, Final Lift - could be done within a one or two hour-long math classes. However, teachers should feel free to adapt this routine to fit their schedules or spend more time on discussion (e.g., using the Mind Stretch as an exit ticket and Workout, Final Lift on the following day). We provide a description of each activity and estimated timeline below for *illustrative purposes*.

Steps	Who?	Purpose?	Est. Time
On Friday			
Set-up	Teacher-led whole class discussion.	Class reviews key ideas, learning goals, expectations and prior knowledge. Set-up can be done on a prior day. For the first task, anticipate that set-up may take longer than the estimated time, while on subsequent tasks, set-up time may be considerably shorter.	5 mins.
Mind Stretch	Whole class.	Students work independently on simple question that requires little or no formal calculations. The focus is on strategies students can use to solve the question. Before starting the Workout, the teacher will share examples of different approaches/representations students could use to help them think, show their thinking.	5 mins.
Workout	Students work independently.	Students practice their procedural skills while also using different representations to explain their approach. Students work independently on a series of related questions with some support/scaffolding.	10-15 mins.
Check your pulse	Pairs, individual, teacher-facilitated whole class discussion	Similar to Think-Pair-Share, partners examine each other's work for similarities and differences. Teacher shares examples with class. Students self-assess their understanding.	10 mins.
Final Lift	Students work independently.	Students apply answers, strategies, ideas from the Workout to a related scenario. Students are given less explicit direction regarding how to solve a problem. More emphasis is placed on using multiple representations to show thinking and/or constructing viable arguments.	10-15 mins.
The following week			
Review w/ colleagues, PLC	Teachers meet virtually.	Teachers collaboratively review student work and discuss instructional strategies/next steps. The FAM "review and reflect" tool can help guide this discussion	60 mins.
On Tuesday or Wednesday			
Class Review/ Reflection	Teacher-led whole class discussion.	Teacher returns student work and provides feedback to students, discusses alternative strategies.	10-15 mins.

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FAM Tasks

Six tasks have been created thus far. We have used existing resources as inspiration, particularly tasks that have been vetted using quality criteria and/or a peer review process.

Task name	Description	Sources of inspiration	Targeted standards
Munching on Math	Students review foundational skills associated with proportional reasoning to predict sales for a new business selling lemonade, brownies, and smoothies.	Ready for Proportional Relationships (p. 72) found in California Go Math! Middle School Grade 8 (Houghton Mifflin/Harcourt), Problem of the Month – Movin 'n Groovin from Inside Mathematics, developed by Noyce Foundation	RP **6th & 7th grade RP standards building blocks for 8EE5-6
By the Pound	Students investigate proportional relationships to get the best deal at the grocery store.	Illustrative Math Tasks, particularly Peaches & Plums task; Coffee by the Pound	8EE.5
Better Jobs	Using multiple representations, students compare proportional relationships & slope for different pay rates.	Illustrative Math Tasks, Better Job,	8EE.5
End of Las Vegas?	Students investigate proportional relationships and match depictions of these relationships in graphs, diagrams, & equations. They apply math skills to thinking about a real-world problem, the drought facing Las Vegas.	MARS tasks, particularly Lines & Linear Equations.	8EE.5
Slippery Slope	Students use similar triangles to explain why the slope is the same between two distinct points on a non-vertical line in the coordinate plane.	Illustrative Math Tasks, particularly the Slopes Between Points on a Lines task, activities from Georgia Department of Education.	8EE.6
Cricket Conversion	Students build on earlier understanding of generating independent sequences and comparing two sequences to one another. They use the concept of ordered pairs to generate a linear equation that will allow them to predict the value of a dependent variable given the value of an independent variable.	Dolbear's law (A. E. Dolbear , "The Cricket as a Thermometer," The American Naturalist 31, no. 371 (Nov., 1897): 970-971.) and the work of Dr. Peggy LeMone. This activity can be expanded to consider the question of how to compare Celsius to Fahrenheit (or vice versa); Other ideas to explore linear relationships (courtesy of jcampbel@wcboe.org)	8EE.6

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Task Materials:

The following materials have been developed to support teachers in their implementation of each FAM task:

1. PowerPoint slides
2. Student worksheets: Handouts for students to work on the problems as they follow along with PowerPoint slides. Of course, students are welcome to use additional sheets of graph paper, etc. as necessary.
3. Answer Key
4. Review and Reflect Tool: A tool for teachers to review student responses in a targeted manner and determine next steps.

**It is important to note that the tasks and supporting materials are *living documents*. The tasks may continue to evolve as we gather more feedback from teachers using them and more examples of student work.

Design Notes for Teachers:

The format of FAM tasks may be somewhat new to you (or very familiar!). In any case, it is important to keep in mind that tasks are intended to:

- Help students learn to manage, direct, and assess their own learning. Teachers should be careful not to provide too many hints or too much direction.
- Provide frequent opportunities for students to receive feedback on their thinking and to assess their work against the criteria. Teachers are highly encouraged to closely follow the task structure and allow students time to ask questions.
- Challenge students to reason, to represent their solutions in multiple ways, and to justify their thinking, specifically in writing. Some degree of struggle is to be expected.
- Deepen their learning of the target standards. Tasks assume that students have been exposed to target standards to some degree (i.e., this is meant to support or review these key concepts, not as primary lessons for these concepts).

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