Introduction

Better Job

COMMON CORE STATE STANDARDS TARGETED:

CCSS: 8EE.5B - Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

CCSS Math Practices:

#1 – Make sense of problems and persevere in solving them.
#2 – Reason abstractly and quantitatively.
#3 – Construct viable arguments and critique the reasoning of others.
#4 – Model with mathematics.
#7 – Look for and make use of structure.
#8 – Look for and express regularity in repeated reasoning.

LEARNING GOALS:[1]

Students will understand/ be able to…

- Make sense of information about proportional relationships from multiple representations.
- Identify the unit rate from given information and represent unit rate using other representations.
- Compare different rates of earning and different starting values.

EXPECTATIONS:

We will know we've accomplished our learning goals when students…

- Gather key information on proportional relationships from graphs, tables, diagrams, and equations and explain that information clearly.
- Determine whether the data in a table can be modeled by a line (i.e., has a constant rate of change).
- State the constant of proportionality (slope) given a table, graph, or equation that represents a line.
- In 8th grade, determine if the table, graph or equation represents a proportion and state the y-intercept.
- Identify the unit rate correctly from given information and represent unit rate using a table, graph and an equation.
- Compare earnings over a certain amount of time based upon given rates of earning, and compare earnings when one rate includes the addition of a constant.

[1] **Inspiration for Task:** Best Job Idea Wave adapted from materials shared by Diane Kinch. Workout and Final Lift tasks have been adapted from Illustrative Mathematics materials, particularly Who Has the Best Job task originally accessed on 5/1/2014, and is licensed by Illustrative Mathematics under CC BY-NC-SA 4.0.

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Across tasks, students should be aware of the following expectations:
Work is accurate and precise:

- The problem is set up in a way that helps you solve it.
- Your responses use appropriate units.
- You have checked your work for calculation errors.

Student explanations:

- Describe what you did and why you did it.
- Use multiple representations to show your thinking about math.
- Include a logical argument and evidence to support each answer. It makes sense.

WHY IS THIS IMPORTANT?

In the elementary grades, students learned to recognize and model simple patterns using mathematical expressions and equations. In the middle grades, this understanding is expanded to include proportional relationships (the notion that two ratios of numbers are equal) between two quantities (variables) of the same or of different units.

One important idea that 6th grade students should grasp during the year is that the ratio 3:4 is the same as the ratio ¾ : 1 (i.e., the unit rate) and begin to connect multiplication and division (e.g., division by a number is mathematically the same as multiplication by the inverse of that number). Seventh grade students will extend this concept to include ratios of fractions and percentages, and to write equations (direct variations) using a "constant of proportionality" (also called a constant of variation). In 8th grade, these concepts form the basis of slope and definition of a linear equation as one with a constant rate of change (slope). Students should realize that not all linear equations include the origin and make the connection between proportional relationships, lines, linear equations, and similarity.

ESTIMATED TIME:

- Approximately 1 classroom period (50 minutes).

MATERIALS:

- PowerPoint Slides
- Student worksheets
- Answer Key
- Graph paper

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WAYS TO MAKE THIS TASK MORE ACCESSIBLE:

- Create proportional relationships that have integer constants of proportionality. For example, in the Workout, make Taylor's wage $20 (rather than $13) for two hours. Ask students to solve this problem and then ask the student to apply the same principles to solve the problem with non-integer solutions. Note: students will often reach the erroneous conclusion that problems must have integer solutions unless they are presented with real world problems that require non-integer solutions. **It is imperative** that students eventually see that not all problems have integer solutions!
- Change the intervals on the Mind Stretch graph to show whole number intervals (rather than intervals that are multiples of five).
- Use smaller numbers. For example, in the Challenge Task, make the CEO's annual salary $360,000 (a smaller multiple of 9) rather than $35,000,000.

WAYS TO EXTEND THIS TASK:

- Include real world situations that can be described by proportional relationships. Possible relationships include: the relationship between centimeters and inches; the height of an object and the length of its shadow; and the diameter of a circle and the circumference of the circle (the constant of proportionality is pi!).
- Show students the tool known as "Dimensional Analysis" or "Unit Cancellation" (also see https://www.youtube.com/watch?v=8jB-LaTGgq8).
- Proportional relationships are often used to estimate population sizes. For example, a sample of 15 bison is taken from a heard of unknown size. These bison are "tagged", released and allowed to disperse back into the heard. The ratio of tagged bison to the total population of the heard is 15:p (where p is the population size). After the tagged bison disperse, another random sample of bison is taken from the heard. Assume that 30 animals are captured and that 3 of these are tagged bison. This proportion is 3:30. If the tagged bison are randomly distributed throughout the entire bison population, then there should be 5 identical groups in the population because $\frac{3}{30}=\frac{15}{p}$, that means the entire bison population (p) should be 5 times 30 or 150 Bison. This relationship can be generalized even further. If the number of tagged bison was allowed to vary and was represented by the variable t, then $\frac{3}{30}=\frac{t}{p}$. This equation can be rewritten as $p=\frac{30}{3}t$ where 30:3 is the constant of proportionality (the number of bison captured to the number tagged). Note that since the equation will **predict** the **total** population **from** the **number tagged**, the constant of proportionality is $\frac{total\ number\ captured}{number\ tagged}$.

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