# Mind Stretch

**Hot & Cold chirping Crickets**

1. Look at how the x values and the y values change in the table below. What would the next x and the next y be if the patterns continue? Fill in the next x and the next y cell.

| X | Y |
|---|---|
| 6 | 4 |
| 9 | 6 |
| 12 | 8 |
| 15 | 10 |
| ⋮ | ⋮ |
| 24 | 16 |

2. How did you decide what to fill in for y when x = 24?
The x column will have a value of 24 three rows below the row where x = 15. The rows after the row where x = 15 will be x = 18, x = 21, and x = 24 (since every x row is three (3) more than the row before it). **That means that there are three rows after the row where y = 10**. These rows will be y = 12, y = 14 and y = 16 because every row adds 2 to the y-value in the row before. So the y-value paired with x = 24 is y = 16.

   The x column is increasing by 3 and the y column is increasing by 2 so 24/3 = 8 and 8*2 = 16 or that y = (2/3)x.



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# Workout

**Hot & Cold** chirping **Crickets**

If the pattern in each column continues, what is y when x = 93?

| X | Y |
| --- | --- |
| 6 | 4 |
| 9 | 6 |
| 12 | 8 |
| ⋮ | ⋮ |
| 24 | 16 |
| ⋮ | ⋮ |
| 93 | 62 |

3. How did you decide what number to fill in for the y value?

   To find the y-value when x = 93, you could just keep adding 3 to 24 to find out how many rows are between 24 and 93.

   The easier way is to use division. First find out how many numbers are between 24 and 93 by subtracting (93 – 24 = 69). Next find out how many groups of 3 are in 69 by division (69/3 = 23). That means you must add 23 groups (rows) of 3 to 24 to get 93. Because x and y are paired, that means you have to add 23 rows to the 16 in the y column to get to the y paired with x = 93. Since every row in y is 2 more than the row before, you must add 2, twenty-three times (or 2 x 23 or 46) to 16. Since 16 + 46 = 62, the y value paired with x = 93 is y = 62.

   You can divide 93 by 3 and then multiply that number by 2 to get y.



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4. How could you write this process as a mathematical rule (i.e., an equation)? *Hint: Remember to use parentheses to tell others in what order they need to perform the operations*.
   Most students will start at the last row (i.e., where x = 24) so the equation will be $\frac{(93-24)}{3} \bullet 2 + 16 = y$; however, students could start at any row, as long as they change the x and y values accordingly. For example, an equation starting from the row where x = 12 and y = 8 would be $\frac{(93-12)}{3} \bullet 2 + 8 = y$.

5. Use your equation to find the value of y when x = 54.
   The equation could be $\frac{(54-24)}{3} \bullet 2 + 16 = 36$. If a student started at a different row, the equation would be different (e.g., $\frac{(54-12)}{3} \bullet 2 + 8 = 36$), but the result will be the same (y = 36).

6. Here is the number sentence that Joan wrote: $y = \frac{(x-6)}{3} \cdot 2 + 4$. Choose an x value in the table and see if Joan's equation gives the correct value of y paired with the x you chose.
   Joan's equation will produce the same value since she just started from a different row. For example, $\frac{(54-6)}{3} \bullet 2 + 4 = 36$.
   Joan started from the top of the table. That is why she subtracted 6 and added 4 (because those were her starting points). She divided by 3 because x is changing by 3. She multiplied by 2 since y is changing by 2.

7. What does each part of the equation **mean**:
   a. $(x - 6)$: This is the difference (or how many numbers there are) between any x value and the number 6 (the number where Joan started).
   b. $\frac{(x-6)}{3}$: This is the number of rows there are between any given x value and the number 6.
   c. $\frac{(x-6)}{3} \cdot 2$: Since every y row is 2 more than the row before it, this will tell you how much you have to add to the y value paired with x = 6 to find the y value that is paired with a given x value. Joan did this to make sure she goes down the same number of rows on x and y.
   d. $\frac{(x-6)}{3} \cdot 2 + 4$: Once you find out how many rows you will move down and multiply the number of rows by 2 (the change in y), you add that to 4 (the y value paired with x = 6) to find the final y value.



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8. Why must slope equal $\frac{change\ in\ y}{change\ in\ x}$ and not the inverse of this ratio?

   Since you know the x value to start with, you divide by change in x to find out how many rows you move on the table. This will be the number of rows you move in the y column as well. You multiply by change in y to find out how much to add to the y value you started with. In all the equations, multiply by 2 and divide by 3 because the amount x and y increase don't change.

9. How could you write an equation directly from the table or a graph of the data table?

   Pick two points on the line. Find the difference between the y-coordinates of the first point and the second point to find the change in y; then the difference between the x-coordinates of the first point and the second point to find the change in x. Use the x-coordinate and y-coordinate of one of the points to start with. Subtract the x-coordinate from x, divide that difference by the change in x, then multiply that entire quotient by the change in y. Finally, add that final product to the y-coordinate. [Students may instead say, find the difference in x-coordinates and the difference in y-coordinates. Divide the difference in y by the difference in x and then multiply by x to find y. Note that this is only true when x and y are in a proportional relationship. This should be made clear to any student who does this!]

## Check Your Pulse

Compare your answers with a partner. Discuss where you agree or disagree.

1. In a few words, explain what part(s) were difficult for you?

Circle the thumb that best describes how you are feeling:

| | | | |
|---|---|---|---|
| | I have lots of questions, I need help.  | Almost got it, but need practice.  | Got it. I can explain this to a classmate.  |



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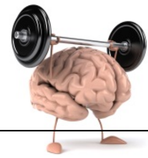
# Final Lift 

**Hot & Cold chirping Crickets**

1. Find the y value that should be paired with x = 21.

| X | Y |
|---|---|
| 3 | 3 |
| 6 | 5 |
| 9 | 7 |
| 12 | 9 |
| ⋮ | ⋮ |
| 21 | 15 |

2. Write an equation that you could use to find the y-value paired with any x.

   An equation could be $\frac{(x-12)}{3} \bullet 2 + 9 = y$, but the student could use any row in the table other than x = 12 and y = 9. [It is important to stress that any row could be used to prepare for the next activity.]

3. Use your equation to find the value of y when x = 93.

   One example of an equation is $\frac{93-12}{3} \bullet 2 + 9 = 63$, but any of the rows can be used to write the equation.

4. Simplify your equation as far as you can.

   The equation can be simplified to $\frac{2}{3}x - \frac{2}{3} \bullet 12 + 9 = y$ or $\frac{2}{3}x - 8 + 9 = y$. The simplest form of the equation is $\frac{2}{3}x + 1 = y$. [Any equation the students write can be simplified to an equation in the form y = mx + b, which is what makes it preferable.]



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Did you know that crickets chirp faster as the temperature goes up? Here is some data that a friend of mine collected one night last summer:

| Chirps (per min) | Temperature (degrees F) |
|---|---|
| 90 | 61 |
| 150 | 75 |
| 165 | 78.5 |
| 180 | 82 |

5. Write an equation that will allow you to predict the temperature outside from the number of cricket chirps you hear in 1 minute. Use x to represent the number of Chirps (per minute) and y to represent the temperature (in degrees Fahrenheit).

   One form of the equation could be $\frac{(x-90)}{60} \bullet 14 + 61 = y$.

6. Simplify your equation as much as possible.

   The following steps could be used to simplify this equation:
   $\frac{14}{60}x - \frac{14}{60} \bullet 90 + 61 = y$ which can be written as $\frac{14}{60}x - 21 + 61 = y$, and this can be simplified to $\frac{14}{60}x + 40 = y$ or $\frac{7}{30}x + 40 = y$

7. Are x and y proportional? Why or why not? Provide evidence for your thinking.

   No! Although this is a line (because the slope is constant), 40 is added to the x term. That means the line does not go through the origin (it goes through (0,40), so x and y are not proportional.



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… but when are we ever going to use this? Let's use your equations to make those crickets tell us the temperature!

You will hear recordings of crickets chirping at different temperatures. *[Note: The length of time on each recording is different, so the number of chirps per minute must be calculated from the number you count. Your teacher will help you with this.]*

8. When listening to the recordings:

- Count the number of chirps in each unit of time
- Estimate the number of chirps you would have heard if the recording would have continued for a whole minute (60 seconds).

| | # of chirps you heard | # of seconds | Estimated # of chirps per minute (these can be provided to the students) |
|---|---|---|---|
| Recording #1 | 23 | 20 seconds | (23 x 3) = 69 |
| Recording #2 | 40 | 20 seconds | (40 x 3) = 120 |
| Recording #3 | 39 | 13 seconds | (39 x 60/13) = 180 |
| Recording #4 | 33 | 8 seconds | (33 x 7.5) = 247.5 |



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9. Now estimate the temperature when each cricket was recorded. Use your estimate the number of chirps in a minute and your equation above (see #1 or #2) to create your estimates.

| | Estimated # of chirps per minute | Calculations using your equation | Temperature (°F) |
|---|---|---|---|
| Recording #1 | 69 | $\frac{14}{60} \bullet 69 + 40$ | 56.1 |
| Recording #2 | 120 | $\frac{14}{60} \bullet 120 + 40$ | 68 |
| Recording #3 | 180 | $\frac{14}{60} \bullet 180 + 40$ | 82 |
| Recording #4 | 247.5 | $\frac{14}{60} \bullet 247.5 + 40$ | 97.75 |

10. Do you think this equation would be accurate for any temperature? Why or Why not? Use some examples to justify your thinking.
It might be too cold or too hot for crickets, so the equation might not be true when the temperature is too cold or too hot. Mathematically, the lowest temperature you could detect would be 40 degrees since there are no such things as "negative chirps". That means x cannot be less than zero. Since the crickets may not chirp at <u>any</u> temperature below 40, the lowest temperature this equation could detect is when x = 1 which is about 40.2 degrees. [Researchers suppose that the lowest temperature at which the crickets chirp is probably around 50 degrees.]



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