Introduction

Hot and Cold chirping Crickets?

COMMON CORE STATE STANDARDS TARGETED:

CCSS: 8EE.5B - Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

CCSS Math Practices:

#1 – Make sense of problems and persevere in solving them.
#2 – Reason abstractly and quantitatively.
#3 – Construct viable arguments and critique the reasoning of others.
#4 – Model with mathematics.
#7 – Look for and make use of structure.
#8 – Look for and express regularity in repeated reasoning.

LEARNING GOALS:

Students will understand / be able to…

- Derive a linear equation from a data set where x and y are in a linear relationship (proportional or non-proportional).
- Explain why slope is defined as change in y (rise) divided by change in x (run) when finding a y-value (output) given an x-value (input).

EXPECTATIONS:

We will know we've accomplished our learning goals when students…

- Can determine if the data in a table, a given graph or a given equation would contain the origin if the pattern in the table continued, the graph was extended or when given an equation of the form y = mx.
- Can determine if a table, graph or equation represents a proportional relationship between x and y.
- Can correctly explain **why** slope is defined as change in y (rise) divided by change in x (run) when finding a y-value (output) given an x-value (input).
- Can correctly state the constant of proportionality (slope) given any representation (i.e., a table, a graph, or an equation) that represents a line and includes the origin.
- Can correctly write an equation (mathematical model) in the form y = mx or y = mx + b given an appropriate set of data in a table or given a clearly labeled graph (i.e., at least two points with (x,y) pairs shown).

Across tasks, students should be aware of the following expectations:

Work is accurate and precise:

- The problem is set up in a way that helps you solve it.
- Your responses use appropriate units.
- You have checked your work for calculation errors.

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Student explanations:

- Describe what you did and why you did it.
- Use <u>multiple representations</u> to show your thinking about math.
- Include a logical argument and evidence to support each answer. It makes sense.

WHY IS THIS IMPORTANT?

In the elementary grades, students learned to recognize and model simple patterns using mathematical expressions and equations. In the middle grades, this understanding is expanded to include proportional relationships (the notion that two ratios of numbers are equal) between two quantities (variables) of the same or of different units.

One important idea that 6th grade students should grasp during the year is that the ratio 3:4 is the same as the ratio ¾ : 1 (i.e., the unit rate) and begin to connect multiplication and division (e.g., division by a number is mathematically the same as multiplication by the inverse of that number). Seventh grade students will extend this concept to include ratios of fractions and percentages, and to write equations (direct variations) using a "constant of proportionality" (also called a constant of variation).

In 8th grade, these concepts form the basis of slope and definition of a linear equation as one with a constant rate of change (slope). Students should realize that not all linear equations include the origin and make the connection between proportional relationships, lines, linear equations, and similarity. Most importantly, rather than just memorizing a definition of slope (e.g., "rise over run"), students should understand **why** slope is defined the way it is when finding a y-value given an x-value.

ESTIMATED TIME:

- Approximately 2 classroom periods (50 minutes each).

MATERIALS:

- PowerPoint Slides
- Student worksheets
- Answer Key

WAYS TO MAKE THIS TASK MORE ACCESSIBLE:

- Use smaller numbers in the tables to make solutions and arithmetic operations easier. *Note, however, that making the difference between rows or the number of rows between entries in the tables too small will encourage students to use simple "add on" counting rather than to model the process mathematically.*
- Ensure that students understand the commutative property of multiplication.
- Help students understand the meaning of fraction multiplication using unit fractions (e.g., $\frac{3}{4} \bullet 4 = 3 \bullet \frac{1}{4} \bullet 4 = 3 \bullet 4 \bullet \frac{1}{4} = 3 \bullet \frac{4}{4} = 3 \bullet 1 = 3$) and integrating the meaning of division and unit fraction multiplication.

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WAYS TO EXTEND THIS TASK:

- Allow the y-intercept (i.e., the y value when x = 0) to have any value.
- Continue x values into negative numbers (e.g., have students model the relationship between degrees Fahrenheit and degrees Celsius, see below).
- Have students explain when negative numbers are and are not appropriate (i.e., justify the domain of the independent variable).
- Show students the tool known as "Dimensional Analysis" or "Unit Cancellation". (Also see https://www.youtube.com/watch?v=8jB-LaTGgq8).
- Proportional relationships are often used to estimate population sizes. For example, a sample of 15 bison is taken from a heard of unknown size. These bison are "tagged", released and allowed to disperse back into the heard. The ratio of tagged bison to the total population of the heard is 15:p (where p is the population size). After the tagged bison disperse, another random sample of bison is taken from the heard. Assume that 30 animals are captured and that 3 of these are tagged bison. This proportion is 3:30. If the tagged bison are randomly distributed throughout the entire bison population, then there should be 5 identical groups in the population because $\frac{3}{30} = \frac{15}{p}$, that means the entire bison population (p) should be 5 times 30 or 150 Bison. This relationship can be generalized even further. If the number of tagged bison was allowed to vary and was represented by the variable t, then $\frac{3}{30} = \frac{t}{p}$. This equation can be rewritten as $p = \frac{30}{3}t$ where 30:3 is the constant of proportionality (the number of bison captured to the number tagged). Note that since the equation will **predict** the **total** population **from** the **number tagged**, the constant of proportionality is $\frac{total\ number\ captured}{number\ tagged}$.
- This activity can be expanded to consider the question of how to compare Celsius to Fahrenheit (or vice versa). The relationship is a linear relationship with the equation F = 9/5·C + 32.
- Ask students how they would find x if they knew a y-value. They will find that "slope" in this case is: "change in x divided by change in y". They should be able to explain this using the same concepts developed in this unit. The students will explore this relationship further when they study inverse functions in high school (F-BF.4)
- Other ideas to explore linear relationships (courtesy of jcampbel@wcboe.org)

Other resources include:

- A discussion of Cricket Thermometers on The Big Bang Theory: https://www.**youtube**.com/watch?v=qCxhS7Aayps
- A video of a chirping cricket: http://www.oecanthinae.com/23206.html
- Digital (.wav) files of chirping crickets: http://entnemdept.ufl.edu/walker/buzz/585a.htm
- See the work of Dr. Peggy LeMone: http://www.globe.gov/explore-science/scientists-blog/archived-posts/sciblog/index.html_p=45.html
- Using Cricket Chirps to Estimate a line of best fit (8.SP): http://blog.minitab.com/blog/the-statistics-of-science/summer-fun-statistics-in-your-backyard-v1
- Other real-world, linear (proportional and non-proportional) relationships from J. Campbel

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TO KEEP IN MIND:

In 8th grade, students will use their understanding of proportionality to mathematically model relationships with linear equations in the form y = mx. Eighth graders will complete their study of lines by modeling situations where the y-intercept of a line can be **any** number and will write equations in the form y = mx + b. An accurate understanding of slope (change in y divided by change in x) will form the basis of understanding linear functions (also in 8th grade), modeling more complex equations (e.g., where the slope is not constant) and functions (including inverse functions).

Understanding slope is also the basis for understanding the Calculus, so an incorrect understanding of slope presents a significant barrier to understanding functions (including inverse functions) and ultimately the Calculus. If **students** have had a lot of practice with proportions they **may immediately write y = ⅔ x** during the first phase of this task. **If** this is **observed**, the teacher should **move to** the discussion of "**Joan's equation**" and what each part of the equation means. Understanding what the part of Joan's equation means is fundamental to the tasks that follow.

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